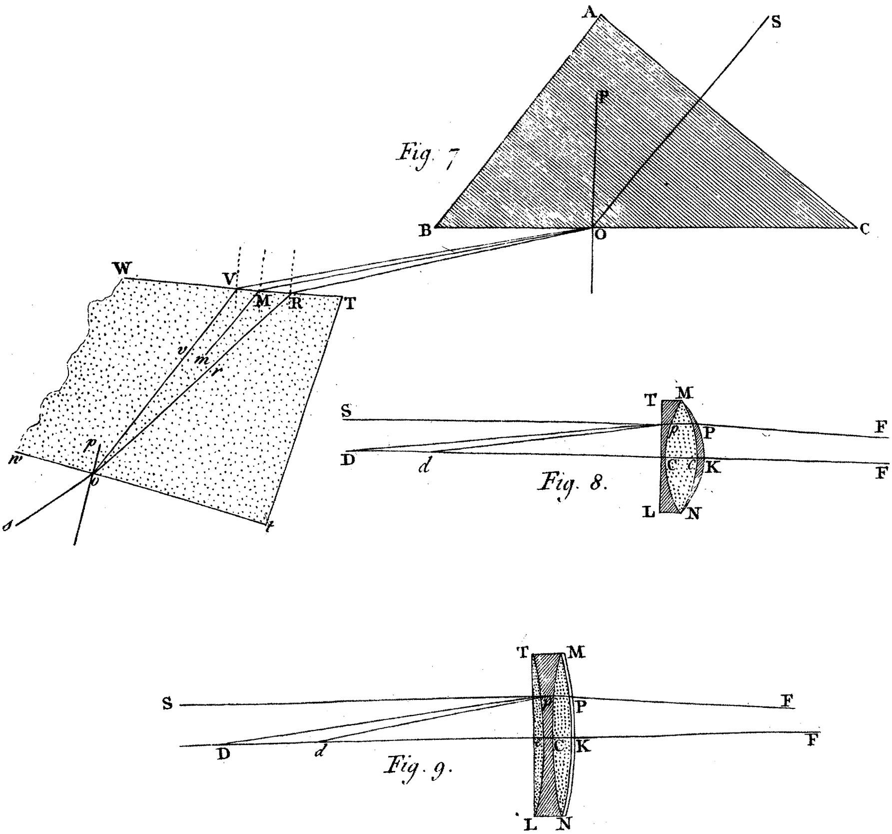
Fig. 7
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Fig. 9
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J. Mynde sc.

XXXI. *Rules and Examples for limiting the Cafes in which the Rays of refracted Light may be reunited into a colourlefs Pencil: In a Letter from* P. Murdoch, *M. A. and F. R. S. to* Robert Symmer, *Efq; F. R. S.* Jan. 3, 1763.

Read June 2, 1763.

1. LET SO, a fmall pencil of the folar light, pafs through the refracting medium ABCD [TAB. XIV. Fig. 1.] whofe oppofite furfaces, reprefented by AB, CD, are parallel planes: then the violet rays OV, will, in the fecond refraction into the air, emerge parallel to the red; for both will be parallel to the incident ray SO, and confequently to each other: that is, V*v* will be parallel to R*r*, as is plain from the common principles of Optics.

2. If the light after its emergence is received on a fcreen placed any where beyond RV, it will be tinged with violet on the fide V*v*, and with red towards R*r*: and if the incident pencil SO is exceeding fmall, all the intermediate colours will be feen in the fame order as when light is refracted by a prifm.

But if the incident pencil is not very fmall; or if the luminous body from which the rays are tranfmitted through a fmall opening at O, has a confiderable breadth, like that of the Sun's difk; then fo many rays of every kind will mix towards the middle of the fpectrum as to produce a pure white; but at the extremities V*v* and R*r*, it will ftill be ting-

ed

ed with violet and red: for a violet ray from the uppermoſt point of the Sun's diſk will be more refracted than the other can be; and a red ray from the loweſt part of his diſk, will be leſs refracted than any other.

3 If BC, the diſtance of the refracting ſurfaces, is increaſed or diminiſhed; RV, the diſtance of the extreme rays, will be increaſed or diminiſhed in the ſame proportion: and if RV approaches very near to the aperture O, the colours will become imperceptible.

4. To reunite theſe rays, we may place another medium of the ſame refractive power, and of the ſame thickneſs ($bc = BC$) as in the figure; ſo as the rays V*v*, R*r*, &c. may enter its ſurface *cd* at the ſame angle as they emerged at from CD, or as SO entered AB; and after refraction at the point *o* of the ſurface *ab*, to which they converge, they will be reunited into *os* the continuation of SO, in a pencil every way like the incident pencil SO, excepting that the light will have been ſomewhat weakened in its paſſage through the media.

5. Other things remaining, let the thickneſs of the ſecond medium be *cp*, leſs than *cb* or CB, the ſurface parallel to *cd* being *pe*; and the emergent rays $\omega\sigma$ will be indeed parallel to the incident as formerly, but the ſpectrum will fall below the place of the ſcreen where SO or *os* would fall. It will likewiſe be coloured, as the rays were not yet united at the point *o*. If the thickneſs be greater than *cb*, the ſpectrum will fall above the line SO *os*, and the violet and red, after their interſection in *o*, will have changed ſides.

6 Other

6. Other things remaining, ſuppoſe the refractive power of the medium *ac* to be increaſed, making the extreme rays to interſect before they reach the ſurface *ab*; in that caſe, let the medium be turned round upon an axis perpendicular to the plane of refraction (repreſented by the plane of the figure) in the order of the letters *a*, *b*, *c*, ſo that the angle of incidence of the rays V*v*, R*r*, the line *vr*, and the angle *vor* may be continually decreaſing till the interſection *o* falls into the ſide *ab*; and the rays will emerge colourleſs and parallel to the incident pencil SO; above, or below, or in the line SO*os*, according to the aſſumed place of the axis of revolution.

If, on the contrary, the refractive power of the medium *ac* be diminiſhed, and, with it, the angle of convergence of the extreme rays; the point where they would interſect falling beyond the ſurface *ab*; the medium muſt then revolve the contrary way, in the order *c*, *b*, *a*; to bring the point of interſection to the ſurface *ab*. But if the refractive power be ſo ſmall that even when *cd* becomes almoſt coincident with V*v*, the point of interſection falls ſtill beyond *ba*, in that caſe the rays cannot be made to emerge colourleſs, otherwiſe than by encreaſing the depth of the medium till its ſurface paſſes through the point of interſection. And in like manner, when the refractive power of the ſecond medium *ac* is greater than that of AC, making the rays to meet within the medium, as at *q* a point in the line *pe*; we may, inſtead of turning the medium round on an axis, cut off the part *pa*, leaving the ſurface *pe* parallel to *cd*; and the emergent light will be colourleſs.

From

From theſe few principles we may determine the phænomena of light tranſmitted through parallelepipids that are contiguous to the air, their poſition and refractive powers being given. Or we may diſpoſe them ſo that the emergent light ſhall, or ſhall not, be tinged with colours.

And we already ſee (what ſhall be more diſtinctly explained below) that if light be tranſmitted through whatever number of media (A, B, C, &c.) all the refractions may be corrected by the equal and contrary refractions of the ſame number of the ſame media (*c*, *b*, *a*,) ſimilar and ſimilarly ſituated to the former; provided there is a medium Z interpoſed between the two ſeries, thus; A, B, C, Z, *c*, *b*, *a*; and that the rays in their paſſage through Z, are parallel to one another.

7. But to give the rays this paralleliſm in their paſſage through Z, and to explain the ſeveral phænomena of refracted light, we ſhall need the following

LEMMA, a PROBLEM.

Given (in Fig. 2.) DCB the difference of two angles ACD, ACB, and the ratio of DI the ſine of the greater to BH the ſine of the leſſer being likewiſe given, to find the angles.

For DF, the ſine of the given difference, write s, and for its coſine CF write c; for the leſſer ſine BH, the letter z, and let the given ratio of DI to BH, be that of m to n, the radius CB being unity.

Then, having drawn FG perpendicular to DI; from the ſimilar triangles in this figure, we ſhall have CB

CB : CH :: DF : DG, or $1 : \sqrt{1-z^2} :: s : DG = s\sqrt{1-z^2}$; and CB : BH :: CF : GI, or $1 : z :: c : GI = cz$. But (by Hypoth.) DI : BH :: $m : n$; that is DG + GI, or $s\sqrt{1-z^2} + cz : z :: m : n$; which gives $\sqrt{1-z^2} : z$, or CH : BH, or 1 : tang. ACB :: $m - nc : ns$; that is, tang. ACB $= \frac{ns}{m-nc}$. In words—multiply the ſine of the given difference by the leaſt term of the given ratio for a dividend: from the greater term ſubtract the product of the coſine of the difference and the leſſer term for a diviſor; and the quotient ſhall be the tangent of the leſſer angle ACB.

Or, if you prefer a geometrical conſtruction; In the ſemidiameter CB produced take CM to CB as DI to BH; and in the tangent to the circle at B, make BL to BC, as DF to FM, and BCL ſhall be the leſſer angle ſought.

Or you need only join DM and draw the ſemidiameter CA parallel to it.

8. But before we apply this ſolution, it may be proper to give a table of the refractive powers of glaſs, water and ſpirit of wine, whether contiguous to the air, or perhaps the fluids contiguous to glaſs: theſe being the ſubſtances in which experiments may be moſt conveniently made: and it is alſo neceſſary to know the limitations that ariſe from thoſe ſeveral powers.

I.

When light passes from air into glass, and the angle of incidence is next to 90°, whose sine is unity;

	°	′	″
The sine of the refraction of the red rays = $\frac{50}{77}$ is .6493508 = sin. - -	40	29	33, 6
And of the violet $\frac{50}{78}$ = .6410256 = sin.	39	52	6
Whose difference	0	37	27, 6

is the greatest angle at which the violet and red rays can diverge in the refraction from air into glass, wanting very little of 37′½.

And when an unrefracted pencil passes from glass into air, as soon as the angle of incidence exceeds 39° 52′ 6″, the violet rays will begin to be reflected; and when the incidence exceeds 40° 29′ 33″, 6 the rays will be totally reflected.

II.

From Air into Water.

	°	′	″
The sine of refraction of the red is .7517905 = s.	48	44	44
Of the violet .7454080 = s.	48	11	39
And the greatest divergence ———	0	33	5

the angle of beginning reflection from water into air being 48° 11′ 39″.

III. From

III.

From Water into Glaſs.

	°	′	″
Sin. incid. : *s.* refr. of the red :: 1 : 0,863739 = *s.*	59	44	20½
Of the violet :: 1 : 0,859966 = *s.*	59	18	45
The difference of which ———	0	25	35½

is the greateſt divergence.

IV.

From Air into Spirit of Wine.

	°	′	″
Sin. incid. : *s.* refr. of the red :: 1 : 0,7334001 = *s.*	47	10	20,2
Of the violet :: 1 : 0,7266366 = *s.*	46	36	18,6
The difference of which ———	0	34	1,6

is the greateſt divergence.

V.

From Spirit of Wine into Glaſs.

	°	′	″
Sin. incid. : *s.* refr. of the red :: 1 : 0,8853964 = *s.*	62	18	0,1
Of the violet :: 1 : 0,8821802 = *s.*	61	54	24
And their difference ——— ———	0	23	36

is the greateſt divergence.

These numbers are partly tranſcribed from Sir Iſaac Newton, and partly computed by a rule of Mr. Euler in the Philaſophical Tranſactions.

They are indeed carried on to more decimal places than the experiments hitherto made can well bear: but it is hoped that hereafter methods may be deviſed to meaſure the refractions of light to a very great degree of preciſion.

9. When a ſlender pencil SO, is refracted by the ſurface of a denſer medium OT (Fig. 3.) the extreme rays being OV, the violet, and OR the red; we have ſeen that the ſurface RVT, at which the rays paſs again into the rarer medium, being parallel to the firſt ſurface OT, the extreme, and all the intermediate, rays will emerge parallel to each other, and to the pencil SO.

But if the laſt ſurface RVT cuts the former in a line perpendicular to the plane of refraction at the point T, on the ſide of the radiant point S, then the extreme rays being refracted at the points V, R, will converge to ſome point F in the rarer medium: and if the light be received on a ſcreen at F, it will be colourleſs; if nearer to the refracting medium, or farther from it, it will be tinged, but on different ſides.

Thus if the denſer medium is water, and the ſurrounding medium is air; the angle of incidence LOS being 20°, the angle of divergence VOR will be 7′ 46″. And OVP the angle of incidence at the ſecond refraction for the violet rays being taken of 30°, the angle of convergence RFY will be 14′ 26″.

On the contrary, if the plane VR*t*, (Fig. 4.) which terminates the denſer medium cuts the firſt refracting plane

plane on the other ſide of the perpendicular OL, the rays will diverge from ſome point *f* on the other ſide of the ſecond ſurface: the violet ray OV being more refracted from the perpendicular VP, than the red is from the perpendicular R*p*.

And it is evident, that if the diſtance (OT or O*t*) of the point of incidence from the edge of a priſm, the angle of incidence LOS, and the angle of the priſm (OTV or O*t*V) are given, together with the refractive powers of the media, the lines OV, OR, will be given in magnitude and poſition. And thence the diſtance VR being given, with the angles of refraction at the ſecond ſurface, the points, F or *f*, to which the rays converge, or from which they diverge, will be given. And their locus, or the Curve in which all theſe points are found, may be aſſigned; whether the angle of the priſm is conſtant, and the angle of incidence is variable, or the contrary; and whether the rays are refracted, or, at a certain obliquity, come to be reflected by the ſecond plane.

10. If it is further required that the extreme, and all the intermediate, rays which meet at F (in Fig. 3.) ſhould thenceforth remain united in a colourleſs pencil: through the point of convergence F draw (by the lemma) the line ZX, making the angles ZFR, ZFV, ſuch that their difference RFV being the given angle of convergence, their ſines may be as the ſines of refraction of the red and violet rays, when they paſs from a given denſer medium GKH into the air, at a common angle of incidence: and HFG perpendicular to ZX will be the line in which the ſurface of that medium muſt cut the plane of refraction, when the rays RF, VF, are refracted into the ſame

ſame line FN. And if the medium be terminated on the other ſide by any plane KN to which FN is perpendicular, the pencil NY, continued in the air, will remain colourleſs.

For inſtance, if the medium GK is glaſs, and the angle RFV is 14′ 26″, ZFR the angle of incidence of the red rays will be found of 17° 54′ 14″; and the angle of refraction XFN, common to all the rays, will be 12° 6′ 34″ $\frac{1}{2}$.

But if the plane HG, to which ZX is perpendicular, paſſes not thro' the point of reunion F, but on this or the other ſide of it; the rays in their paſſage thro' the medium, though parallel to each other, will be laterally ſeparated.

11. Let a ray SOL (Fig. 5.) of a mean degree of refrangibility be refracted by AB the ſide of a glaſs priſm ABC, ſo that the refracted ray OM may be perpendicular to the ſide of the priſm AC; it is required to apply to this another priſm of a differently refracting ſubſtance, as of water, ſo that the ray M*o* being refracted at *o*, by the ſide DC, the refracted ray *so* may be parallel to OS.

The angle of incidence SOP, and the refractive power of the glaſs being given, the angle SOM, and its ſupplement LOM, are given produce M*o* to *n*; and becauſe *os* is to be parallel to LO take for the difference of the angles in the lemma, the given angle *nos* (=LOM), and through the point *o* draw *rop*, ſo that the ſine of *pon* may be the ſine of *pos*, as the ſine of incidence to that of refraction, when a meanly-refrangible ray paſſes from water into air; and D*o*C, perpendicular to *rp*, will be the poſition of the ſide required.

We have here ſuppoſed the ray SO to be homogeneous, of a mean refrangibility; but if it is a ray from the Sun the image at *s* will be very much tinged. The colours will have been ſeparated at O; a ſmall matter more at M, but they will diverge very conſiderably at *o*; for ſetting aſide the refractions at O and M; that is, ſuppoſing a pencil M*o* to paſs unrefracted in water till it falls upon a ſurface of air at an angle of incidence of about 47° 32′$\frac{1}{2}$, the divergence of the extreme rays will be about 2° 51′$\frac{1}{2}$: a ſmall difference of ſines anſwering to a conſiderable difference of the angles when they approach to 90°: the ultimate difference to which they converge, being (from water into air) 7° 26′$\frac{1}{4}$.

12. Let a pencil of the ſolar light SO (Fig. 6.) fall upon the ſurface of water BC, the extreme rays being refracted into OV, OR; it is required to aſſign the glaſs priſm PN*n* (whoſe ſection PN*n* is an iſoſceles triangle) ſuch, that the baſe N*n* being parallel to SO, and the ſurface of the water AC being inclined to the baſe N*n* in the ſame angle as the ſurface BC; the extreme rays, in their paſſage through the glaſs priſm, ſhall be parallel; and all the rays ſhall emerge colourleſs in the line SO*os*; that is, in the incident ray produced thro' both the media.

The angle SOB, and the refractions from air into water, being given, the angles VON, RON, and their difference VOR, are given. Draw therefore, by the lemma, the line OG, making the ſine of ROG to that of VOG, as the ſine of refraction of a red ray, in paſſing from glaſs into water, is to the ſine of refraction of a violet ray, their angles of incidence being equal, and PN perpendicular to OG will be the interſection of of the plane of refraction with the ſide of the priſm that is required.

Thus the angle SOB being 30°, VOR will be 18′ 12″$\frac{1}{2}$, VOG=50° 38′ 4″$\frac{1}{2}$, ROG=50° 19′ 52″$\frac{1}{2}$. whoſe ſines are as the ſines of refraction of the violet and the red, in paſſing from glaſs into water at a common angle of incidence. And therefore, the angles of the emergence of the rays OV, OR, in paſſing from water into glaſs will be equal, that is V*v* will in its paſſage through the glaſs priſm, be parallel to R*r*, and the rays meeting with equal and contrary refractions at the points *v*, *r*, *o*, as they ſuffered at V, R, O, will emerge colourleſs at *o*.

Yet we muſt not be ſurprized if the pencil *os* is not abſolutely pure light (even ſuppoſing, the matter, the figure, and the diſpoſition, of the media to be faultleſs) becauſe (1°) perhaps the refractive powers have not been determined with ſufficient exactneſs (2°). If the glaſs plate which contains the water be not very thin, the light will have received a ſlight tincture in paſſing through it at O: This however may be remedied by confining the water between two glaſs priſms. And (3°) it its ſcarce poſſible to make experiments of this kind with a pencil of light ſo ſlender as the theory preſcribes (ſee § 2.)

But proper allowances being made on theſe accounts, and the refracting planes adjuſted as the lemma directs, the light will emerge ſufficiently pure to juſtify the theory. And the refractions of either medium being given, it will appear from the experiments whether thoſe of the other medium have been determined with ſufficient accuracy.

Obſerve

Obſerve likewiſe, that as, in practice, we muſt fit the water to the glaſs, not the glaſs to the water, we are to begin by aſſuming VR of a convenient magnitude; and ſuppoſing the rays V*v*, R*r*, &c. to be parallel within the glaſs, find the point O to which they converge in the water, through which a plane may be drawn which ſhall ſend them out into the air, in a colourleſs pencil OS.

R E M A R K S.

I.

The 8th experiment in Sir Iſaac Newton's optics (Book I. Part 2.) ſeems to have been made under the conditions which are limited by the foregoing problem; though he does not ſpecify theſe conditions. For, it is to be preſumed, he did not combine his priſm and water at random, but adjuſted them ſo as to produce the expected effect. It is obſerved likewiſe, that he does not give us a deſcription of his experiment ſo particular as, in moſt inſtances, he was wont to do. He thought perhaps that the conſequences he deduces from it might ſufficiently explain his meaning; eſpecially as he had, in the foregoing propoſitions, fully eſtabliſhed the principles of his theory.

However this be, ſeveral perſons of ſkill and addreſs in optical matters, have produced experiments in contradiction to that of Sir Iſaac, and have affixed meanings to his concluſions which he never could intend, without being groſsly inconſiſtent with himſelf: an imputation from which common candor and decency ought to have protected ſo great a name *.

For inſtance, when he ſays that " light as often as by contrary refractions it is ſo corrected that it emergeth in lines parallel to thoſe in which it was incident, continues ever after to be white"; can this aſſertion poſſibly bear the meaning they would obtrude upon us? Had Sir Iſaac ſo entirely forgot his own doctrine as not to know, That if the glaſs priſm PN*n*, in the laſt ſcheme, is, any where above V*v*, terminated by a plane to which the pencil SO is perpendicular, the rays V*v*, R*r*, &c. though emerging parallel to SO, will exhibit their ſeveral colours? The ſenſe therefore which the experiments affix to Sir Iſaac Newton's words being ſo abſurd, had not they done better to look out for one that was conſiſtent with his theory? and ſuch a one they would have found by only drawing a figure like the foregoing; where the rays of the pencil, reunited in *os*, as well as when ſeparated within the glaſs priſm, are parallel to each other and to the incident pencil. But, if the water is terminated by a plane different from AC, paſſing through the point *o*, and making the rays (no longer parallel to SO) to diverge, then the light will, by degrees, in paſſing on from *o*, become coloured: which is Sir Iſaac's other poſition.

To this meaning his own words ought to have led the objectors. It was light, not ſeparate rays, which

* The reader ought to be told, that it is not here intended to detract from the merit of the late Mr. Dollond's improvement of refracting teleſcopes; but only to correct a miſtake of his concerning that difference of diſperſion of rays, which he has ſo happily applied to uſe.

emerged

emerged in his experiment; and which (being parallel to the incident light) continued to be colourless.

He adds farther, " the permanent whiteness argues, that in like incidence of the rays, there is no separation of the emerging rays": as much as to say, that in his experiment (as in our 6th Figure) the pencil, in passing or repassing, is supposed to meet with surfaces of equal refractive powers, similarly situated.

The other cases in which refracted light may recover its whiteness, although it emerges not parallel to the incident, or may be tinged though parallel to it, Sir Isaac does not treat of: the experiment he had made, being sufficient for the purposes to which he applies it. But he assures his readers, that if they will argue truely upon his theory, trying all things with good instruments, and sufficient circumspection, the expected event will not be wanting. And the fact is, that in all the experiments which have been made, if none of the necessary data are wanting, the appearance of the emerging light may be certainly prodicted.

II.

When a slender pencil of light is refracted at the surface of any medium, the extreme rays, the violet and red, and the several intermediate rays, each of its particular degree of refrangibility, will all diverge from, or converge to, the same physical point: or when that point, by altering the position of the plane, is thrown to an infinite distance, will all of them become parallel. And it appears from the foregoing solution, that such parallelism may always be effected,

 whatever

whatever be the refracting power of the medium PN*n*, provided that, in a given medium, the quantities *m*, *n*, &c. of the lemma, which represent the sines of refraction of the several sorts of rays, to a common sine of incidence, continue to be in constant ratios to one another.

Conversely, if, from experiments such as that which Sir Isaac Newton made, it follows that, whatever be the refractive powers of the media, and the angle of incidence of the light, the pencils SO, *so*, may be made to reciprocate with each other, while all the sorts of rays, in passing or repassing through the prism PN*n*, become parallel; if, I say, this is confirmed by experiments, it is a proof that, for any given medium, the ratios of those quantities *m*, *n*, &c. are invariable.

III.

And hence Sir Isaac deduces the two theorems subjoined to his 8th experiment; by the first of which he contrives to make the ratios of the sines of refraction belonging to the several sorts of rays, to a common sine of incidence, when they pass from glass into air, to serve for finding the like ratios for the rays passing from water into air, without the trouble of new experiments.

His first theorem may be deduced in this manner:

Let all the sorts of rays, whether united in a pencil of light, or separated parallelwise by refraction, have the same angle of incidence whose sine is I, when they pass from a denser into a rarer medium; and let V and R stand for the sines of refraction of the

the extreme (or any two ſorts of) rays. Then ſeeing by the experiments, the ratio of V to I is given, as alſo that of R to I; the ratio of V—I to I, as alſo (invert.) that of I to R—I, and (exæquo) that of V—I to R—I, are given: for this laſt write the ratio of 1 to p.

In like manner, let the refractive power of the medium from which the rays emerge into the ſame medium as before, be increaſed or diminiſhed, as alſo the common angle of incidence; and we need only write other marks v and r for the ſines, and i for the common ſine of incidence; for we ſhall have as before $v-i$ to $r-i$ in a given ratio; which call that of 1 to q. And, from theſe two, we have $\frac{V-I}{v-i} = \frac{R-I}{r-i} \times \frac{q}{p}$. But p is always nearly equal to q; in the refractions from glaſs, and from water into air, their difference is leſs than $\frac{1}{400}$ part of either; we may therefore put the ratio $\frac{V-I}{v-i}$ equal to $\frac{R-I}{r-i}$; which is the firſt theorem.

And thence, if one difference R--I becomes equal to r--i, the other differences V--I, &c. will be reſpectively equal to v--i, &c. and the ſame ſet of differences may be made to ſerve for ſeveral media, provided the ſines of incidence are taken in their due proportion.

Thus when red rays paſs from glaſs into the air, we have I : R :: 50 : 77 and R--I : I :: 27 : 50, and when they paſs from water into air $i : r-i$ :: 3 : 1, and therefore, as we are to make R--I every where equal to r--i, we get, ex æquo, i : I :: 81 : 50, as Sir Iſaac Newton finds it.

IV. But

VI.

But to explain this matter a little farther, and obviate ſome difficulties concerning it, I ſhall add the following

E X A M P L E S.

The refractive powers being as marked above, let red rays fall from glaſs into air at the angle of incidence 20°, the angle of refraction will be 31° 47′.

Again, let them fall from water into air at an angle of 34° 1′, making their angle of refraction 48° 5′.

And the difference of the ſines of 31° 47′, and 20° will be preciſely equal to the difference of the ſines of 48° 5′, and 34° 1′.

At the ſame angles of incidence 20° and 34° 1′, let the violet rays fall from glaſs and water into air; and the angle of refraction from the glaſs, will be 32° 14′$\frac{1}{2}$ nearly, and that from the water will be 48° 38′ nearly, And the difference of the ſines of 32° 14′$\frac{1}{2}$ and 20° will equal the difference of the ſines of 48° 38′ and 34° 1′, within .000488, or leſs than $\frac{1}{2000}$th part.

We ſee likewiſe that the red and violet rays diverged from the glaſs medium at an angle of 27′$\frac{1}{2}$; but from the water at an angle of 33′; making the difference of divergence in this example 5′$\frac{1}{2}$; that is $\frac{1}{5}$ of the whole divergence of the red and violet rays when refracted from glaſs into air, at incid. 20°.

Whence it appears, that although the differences of ſines above ſpecified, or the exceſſes in Sir Iſaac's theorem, may, in refractions from different media into the ſame rarer medium, be made equal, it does by no means follow, that the divergences of the ſeveral ſorts of rays (or if you chuſe to call it their diſperſion) will be the ſame in the two refractions; for Sir Iſaac's exceſſes 27, $27\frac{1}{9}$, &c. are the exceſſes of ſines; not of angles, as ſome opticians ſeem to have miſapprehended.

Again, let an unrefracted pencil of light fall from common glaſs into the air (Fig. 7.) at the incidence 39°, and the angles of refraction will be,

	°	′	″
Of the violet - - - -	79	2	2
Of the red - - - - -	75	43	55

And their difference - 3 18 7 is the divergence of the extreme rays.

And the angle of refraction of the mean ray is 77° 16′ 19″.—By *mean ray* is underſtood the ray whoſe ſine of refraction is a geometrical mean between the ſines of refraction of the extreme rays, the common radius being unity.

Let now the ſame rays be refracted the contrary way by a ſurface of water WT, then, to make the mean emergent ray parallel to the incident pencil, its angle of incidence muſt be 86° $37'\frac{2}{3}$: and the extreme rays will now converge at an angle of $20\frac{1}{8}$ minutes, nearly.

Through the point of convergence *o*, draw (by the Lemma) a plane *wt*, to terminate the water, and unite all the rays into a colourleſs pencil *os*: and this emergent

emergent pencil will be found to make with a perpendicular to the terminating ſurface an angle of 49° 6′$\frac{1}{2}$, and will be inclined to the firſt incident pencil in an angle of 14 degrees, 28 minutes, 20 ſeconds. Nor is there any other plane beſides this which will thus unite the rays. If planes parallel to it cut the rays any where but in their point of convergence, they will be parallel to each other, but exhibiting their ſeveral colours. And planes not parallel to it, will every where give a coloured image, excepting only when they paſs through the point of convergence; but then the rays having croſs'd at at that point, will thenceforth diverge from one another, and give a coloured ſpectrum.

From all which it appears that light refracted thro' different media may emerge colourleſs, although its firſt direction be conſiderably altered. And that its mean direction may remain the ſame, though its extremities be ſenſibly tinged with colours. Poſitions which, I know not by what miſhap, have been deemed paradoxes in Sir Iſaac Newton's theory of light, but which are really the neceſſary conſequences of it.

Of Teleſcopical Object-Glaſſes giving an Image free from Colours. Fig. 8 and 9.

If the extreme rays, the red and violet, after one or more refractions, diverge from points D and *d*, the diſtance of the point of divergence of the leaſt refrangible from the lens, being greater than that of the moſt refrangible, ſuch a ſemidiameter of the laſt ſpherical ſurface, from which they are to paſs into the

the air, may be aſſigned, as ſhall unite the extreme, and all the intermediate, rays in the ſame focus F; neglecting the aberration from the figure.

The RULE is this;

For the diſtances of the points of divergence from the lens, write D the greater, and d the leaſt; the ſemidiameter of any of the given ſurfaces being aſſumed for unity: And $\frac{M}{r}$, $\frac{m}{r}$ expreſſing the ratios of the ſines of incidence and refraction of the violet and red rays out of air into the laſt medium whoſe ſurface is required: the ſemidiameter of that ſurface will be $\frac{\overline{M-m} \times Dd}{MD-md}$; as may be eaſily demonſtrated from a theorem of Dr. Smith, in the remarks ſubjoined to his Optics.

Thus if the laſt medium is glaſs, the ſemidiameter of the ſurface from which the rays paſs into the air, muſt be $\frac{Dd}{78D-77d}$, it being, in this caſe, $\frac{M}{r}=\frac{78}{50}$, $\frac{m}{r}=\frac{77}{50}$.

EXAMPLE I.

Let MpCNcM (Fig. 8.) be a double convex lens of water confined between the plano-concave MTLN, and the meniſcus MKNcM, both of glaſs, and having the radii of their ſurfaces contiguous to the water, equal to each other, or to unity: and if a ray Sp, parallel to the common axis of the lenſes, after being refracted by the aqueous lens, have its

 extreme

extreme rays, the red and violet, divergent from the points D and *d*; the diſtance of F, the focus where all the rays can meet, will be 8.898: and when this happens, the exterior ſurface of the meniſcus, that is, the ſurface repreſented by MPKN, will have its radius to that of the inner ſurface M*c*N, as 139 to 154.

Example II.

When a double concave of glaſs (the radii of whoſe ſurfaces are unity) is incloſed in water, as in Fig. 9, the water being confined on one ſide by a thin glaſs plate TL, and on the other by a concentrick ſpherical ſhell MPKN; the ſemidiameter of this ſhell muſt be to unity as 471 to 547: and the focal diſtance CF, at which the colourleſs image is formed, will be $4.77\frac{2}{3}$. In theſe examples the thickneſs of the lenſes is neglected; but it may eaſily be taken into the account, if it is thought neceſſary.

The ſame thing may be effected by means of any media of different refractive powers: for the ſemidiameter of the laſt refracting ſurface being determined according to the foregoing rule, the nearer diſtance of the points of divergence (*d*) of the more refrangible rays will be ſo compenſated by their greater refrangibility, that all the rays will converge to the ſame focus F. And this without introducing any new principle into the ſcience of optics, or any diſperſion of light different from the refractions diſcovered by Sir Iſaac Newton near a hundred years ago.

XXXII. *An*

Note *The Figures Serving only for a general Illustration are not Geometrically Exact.*